Multi-site precipitation downscaling via an expanded conditional density network

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Outline

1. Downscaling
2. Neural network vs. Density Network
3. Precipitation
4. Multi-site precipitation
5. Expanded downscaling
6. Expanded Poisson-gamma density network
7. Application to stations in Kootenay region
Climate Scenarios & the Need for Downscaling

Global Climate Model (GCM)
\[\sim 100-1000 \text{ km}\]

Impact Assessments
\[\sim 1 \text{ km or at station(s)}\]

Mismatch in Scales
Solution - Downscaling

Global Climate Model

Downscaling

⇒ Develop intermediate downscaling model to bridge the gap in scales

1. Dynamical model
2. Statistical model
   - linear regression
   - neural network etc.
Neural Network versus Density Network

e.g.,

conditional mean, constant variance (homoscedastic)

conditional mean & variance (heteroscedastic)

→ normally distributed variables like temperature
Properties of Precipitation – Implications for Modelling

- discontinuous in space and time
- non-Gaussian distribution

→ separate models for (1) precipitation occurrence,
  (2) transformed amounts
→ application to multi-site precipitation?
Problem - Spatial Relationships Between Stations

Even with a multi-site model:
- inconsistency between downscaled and observed spatial relationships between stations

Implications for watershed modelling:
- events may be too wet/dry, patterns/_gradients wrong
Solution: Change the criterion used to calibrate the model

Downscaling $\rightarrow$ Expanded Downscaling (Bürger, 1996)

$$C = \sum_{i=1}^{N} \sum_{k=1}^{K} (y_k(t) - y_k^{obs}(t))^2 + \alpha \sum_{i=1}^{K} \sum_{j=i+1}^{K} (\text{cov}(y_i, y_j) - \text{cov}(y_i^{obs}, y_j^{obs}))^2$$

Error term

Covariance constraint

Preserves linear relationships between stations
Putting It All Together

1. Probability distribution (discrete & continuous)
2. Density network model
3. Expanded downscaling calibration criterion

Probabilistic Multi-site Precipitation Downscaling
1) Poisson-gamma Probability Distribution

(a) Poisson-gamma pdf with $\mu = 3.5$ and $\phi = 4.5$

(b) Inverse Poisson-gamma cdf with $\mu = 3.5$ and $\phi = 4.5$

Model precipitation occurrence & amount at the same time
2) Poisson-gamma Density Network

\[ x_i \]

\[ w_{ij}^{(1)} \]

\[ h_j \]

\[ w_{jk}^{(2)} \]

\[ O_k \rightarrow \mu = \exp(o_1) \quad \text{(mean)} \]

\[ \rho = 1 + \frac{1}{1 + \exp(o_2)} \quad \text{(power)} \]

\[ \phi = \exp(o_3) \quad \text{(dispersion)} \]

Fit via maximum likelihood
Using a Density Network to Predict Precipitation

Density Network

\[ x(t) \rightarrow [\mu(t), \rho(t), \phi(t)] \rightarrow y(t) \]

Sample from Distribution

Model Inputs

Poisson-Gamma Parameters

Precipitation (mm)

\[ \text{pdf} \]

\[ \text{cdf} \]

\[ y (\text{mm}) \]

\[ z (\text{cum. prob.}) \]
e.g.,

Observed Precipitation

Precipitation (mm)

Time (days)
e.g.,

Conditional quantiles ($z=0.05, 0.50, \& 0.95$)
Observed occurrence (N=690)

- Time (days): 0 to 1000
- Occurrence of precipitation: 0.0 to 1.0

Example (cont.)
e.g. (cont.)

Predicted occurrence (N=692)

Occurrence of precipitation

Time (days)
3) Expanded Poisson-gamma Density Network

Expanded downscaling for multi-site precipitation in a probabilistic framework:

\[ C_1 = -\sum_{i=1}^{N} \sum_{k=1}^{K} \ln P(y_{k}^{obs}(t) \mid x(t), \mu(t), \phi(t), \rho(t)) + \alpha \sum_{i=1}^{K} \sum_{j=i+1}^{K} \left( \text{cov}(y_i, y_j) - \text{cov}(y_i^{obs}, y_j^{obs}) \right)^2 \]

**Negative Log-Likelihood**

\[ C_2 = -\sum_{i=1}^{N} \sum_{k=1}^{K} \ln P(y_{k}^{obs}(t) \mid x(t), \mu(t), \phi(t), \rho(t)) + \alpha_1 \sum_{i=1}^{K} \sum_{j=i+1}^{K} \left( \text{cor}(\mu_i, \mu_j) - \text{cor}(y_i^{obs}, y_j^{obs}) \right)^2 \]

\[ + \alpha_2 \sum_{k=1}^{K} \left( <\phi_{k}\mu_k^{\rho_k}> + \text{var}(\mu_k) - \text{var}(y_k^{obs}) \right)^2 + \alpha_3 \sum_{k=1}^{K} (\mu_k - <y_k^{obs}>)^2 \]

**Covariance constraint**

**Modified constraints required for computational reasons**
Test – Daily Precipitation in Kootenay Region

Wasa
Fernie
Baynes Lake Kootenay River
Cranbrook SE
Wardner Kootenay Hatchery
Model Calibration & Testing

1) Inputs → synoptic-scale surface circulation, mid-troposphere circulation, & low-level moisture fields; outputs → daily precipitation at five stations


3) Comparison between "expanded" (results in red) and "regular" (results in blue) multi-site downscaling algorithms
Test Results – Station Correlations & Covariances

(a) EPDN–NC
(b) EPDN

Correlations

Regular

Expanded

Covariances
Test Results – Quantile-Quantile Plot of Regional Precipitation

(a) EPDN-NC

(b) EPDN

Regular

Expanded
Conclusions

1) Poisson-gamma density network can predict the conditional distribution of multi-site precipitation at a daily time step

2) Prediction parameters of the Poisson-gamma distribution allows precipitation occurrence and precipitation amounts to be modelled at the same time

3) Principles of expanded downscaling can be applied in a probabilistic modelling framework to allow realistic representation of spatial relationships between stations